Short Note

The ellipticity of Rayleigh waves at infinite depth

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Resumen

Se presenta una fórmula analítica y una aproximación para calcular la elipticidad de las ondas de Rayleigh en un semi-espacio homogéneo a profundidad infinita, en función del módulo de Poisson. Se compara el resultado con las fórmulas correspondientes para elipticidad en la superficie.

Palabras clave: Ondas de Rayleigh, elipticidad.

Abstract

I present an analytical formula and an approximation for the ellipticity of Rayleigh waves in a homogeneous half-space at infinite depth in terms of Poisson's ratio. The result is compared with the corresponding formulas for surface ellipticity.

Key words: Rayleigh waves, ellipticity.

Introduction

The ellipticity or H/V ratio χ_0 of seismic Rayleigh waves propagating on the surface of the Earth has attracted the attention of experimental seismologists (see, e. g., Lermo and Chávez-García, 1994; Bard, 1998); Flores-Estrella, 2004) as well as theoreticians (see, e. g., Malischewsky and Scherbaum, 2004). The ellipticity of Rayleigh waves at infinite depth χ_∞ is a significant parameter that belongs to a complete theoretical description of the wave field. Weichert (2007) pointed out that the ellipticity adopts a constant value at infinite depth. This is indeed the case: its value may be simply determined as a function of Poisson's ratio ν. It should be noted, however, that the meaning of "infinite depth" is frequency-dependent. It can extend over an interval of many kilometers for long-period Rayleigh waves, or of a few meters for very high-frequency waves. This feature can be significant in some geophysical situations, e. g. for borehole measurements.

An analytical formula for χ_∞

Representations of the Rayleigh eigenfunctions for a homogeneous half-space lead to the simple formula

$$\chi_\infty(\nu) = \sqrt{1-x(\nu)}\,, x=c^2/\beta^2, \tag{1}$$

where c is the phase velocity and β is the shear-wave velocity. By using Malischewsky's formula for the Rayleigh-wave velocity in a half-space (see Malischewsky, 2004), the ellipticity at infinite depth may be expressed analytically as a function of Poisson's ratio as

$$\chi_\infty = \sqrt{1-2g_4(\nu)} \tag{2}$$

where the following abbreviations are used:

$$g_3(\nu) = 17 + 3\sqrt{33 - 24\bar{\nu}^3 + \frac{321}{4}\bar{\nu}^2 - 93\bar{\nu}} - \frac{45}{2}\bar{\nu}\,,$$

$$g_4(\nu) = \frac{1}{3}\left[4 + \frac{2(1-3\bar{\nu})}{\sqrt[3]{g_3(\nu)}} - \sqrt[3]{g_3(\nu)}\right], \text{ and}$$

$$\bar{\nu} = 1 - \frac{\nu}{1-\nu}\,, \tag{3}$$

and the main values of the cubic roots are used throughout. For the reader's convenience we recall the formula for χ_0 (see Malischewsky *et al.*, 2007):

$$\chi_0 = \frac{\sqrt{1-2g_4(\nu)}}{1-g_4(\nu)} = \frac{\chi_\infty}{1-g_4(\nu)} \tag{4}$$

The ellipticities χ_0 and χ_∞ in terms of Poisson's ratio are shown in Fig. 1 together with the difference between both values. Negative Poisson's ratios do not arise in seismology, except in some particular crystallization phases of ice (Bormann, 2002). However, they do have some importance in material science, and they are included here for completeness. At infinite depth, the ellipse described by particle motion is always flatter than it is on the surface. In Fig. 1, the difference between the ellipticities has a maximum at $\nu = 0$ ($\chi_0 - \chi_\infty = 0.3$), and the relative deviation $(\chi_0 - \chi_\infty)/\chi_0$ is maximum at $\nu = 0.5$. In the valley of Mexico, Poisson's ratio is as high as 0.499 and the ellipticity at infinite depth reaches 54.4% of the surface ellipticity.

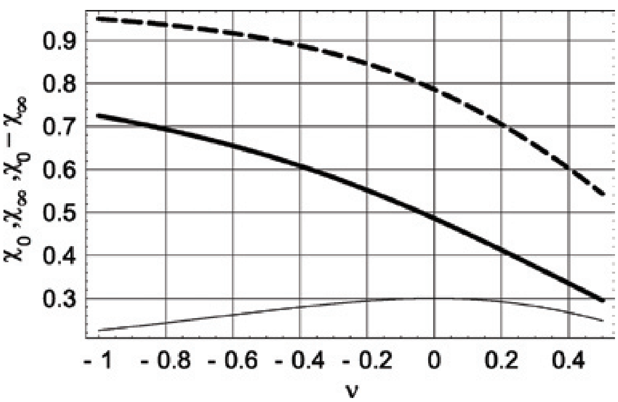

Fig. 1. The ellipticities χ_∞ (heavy line), χ_0 (dashed line) and χ_0-χ_∞ (thin line), as a function of Poisson's ratio ν.

Finally, we may make use of a method proposed by Pham Chi Vinh and Malischewsky (2006) which involves carrying out a Taylor expansion of Equations (2) and (4) in the interval $\nu \in (0,0.5)$. The approximation is very accurate, with a relative error of less than 0.1 % in the whole interval:

$$\chi_\infty = 0.486 - 0.355\nu - 0.080\nu^2 + 0.057\nu^3 ,$$
$$\chi_0 = 0.786 - 0.349\nu - 0.292\nu^2 + 0.041\nu^3 . \quad (5)$$

These formulas may be easily inverted to obtain Poisson's ratio. Let

$$m_1(u) = \left[-0.027 + 0.0877\left(u + \sqrt{(u+0.0881)(u-0.7046)}\right)\right]^{1/3} \quad (6)$$

$$m_2(u) = \left[0.0517 + 0.0454\left(u + \sqrt{(u+3.1621)(u-0.8829)}\right)\right]^{1/3} ; \quad (7)$$

then the equations for the corresponding ellipticities are

$$\nu = \mathrm{Re}\left[0.4678 - \frac{0.2472 - 0.4282\,i}{m_1(\chi_\infty)} - (2.3208 + 4.0197\,i)\,m_1(\chi_\infty)\right] , \quad (8)$$

$$\nu = \mathrm{Re}\left[2.374 - \frac{0.6565 - 1.1372\,i}{m_2(\chi_0)} - (3.2264 + 5.5883\,i)\,m_2(\chi_0)\right] , \quad (9)$$

where i denotes the imaginary unit and the main values of the cubic roots should be used. These formulas may also be useful in possible new applications of the *H/V*-method for non-destructive testing (see Malischewsky *et al*., 2006, and Weichert, 2007).

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